BULETINUL INSTITUTULUI POLITEHNIC DIN IAŞI
Publicat de
Universitatea Tehnică „Gheorghe Asachi” din Iaşi
Volumul 65 (69), Numărul 2, 2019
Secţia
MATEMATICĂ. MECANICĂ TEORETICĂ. FIZICĂ

ACOUSTIC SCATTERING-ABSORPTION CROSS SECTION OF ELECTROSTATIC TYPE

BY

ION SIMACIU [1,*], GHEORGHE DUMITRESCU[2], ZOLTAN BORSOS[1], ANCA BACIU[1] and NAN GEORGETA[1]

[1]Petroleum-Gas University of Ploieşti, Romania
[2]High School Toma N. Socolescu, Ploieşti, Romania

Received: May 14, 2019
Accepted for publication: June 7, 2019

Abstract. The analysis of the secondary Bjerknes force between two bubbles suggests that this force is analogous to the electrostatic forces. The same analogy is suggested by the existence of a scattering-absorption cross section of an acoustic wave on a bubble. In this paper we analyze the analogy between the scattering–absorption (extinction) cross section of an acoustic wave by a bubble into the fluid and scattering–absorption cross section of electromagnetic waves by an electric charge.

Keywords: secondary Bjerknes force; acoustic force of electrostatic type; acoustic cross section.

1. Introduction

In this paper we study the scattering–absorption (extinction) cross section of the acoustic waves on a bubble that oscillates in the liquid. The aim is to highlight the analogy between the acoustic and electrostatic interaction (Simaciu *et al*., 2019).

The scattering-absorption phenomenon of the acoustic wave involves a cross section of interaction (Ainslie and Leighton, 2011; Prosperetti, 1977).

[*]Corresponding author; *e-mail*: isimaciu@yahoo.com

The study of the two phenomena leads us to a better phenomenological understanding of the microscopic phenomena of the electromagnetic world, *i.e*. the world in which systems and phenomena interact and correlate with electromagnetic waves.

2. The Scattering and Extinction Cross Section

2.1. The Acoustic Scattering and the Extinction Cross Section

The acoustic cross section is analogous to the interaction cross section of an electrically charged oscillator with the field of the electromagnetic waves. Hence we will infer some correspondences between the acoustic and the electromagnetic parameters.

The interaction cross section is defined as the ratio of power (scattered and absorbed by the system) and the intensity of the incident wave (Prosperetti, 1977)

$$\sigma = \frac{P}{I}. \tag{1}$$

For the acoustic waves which interact with a bubble and hence performing radial oscillations in a fluid, the power absorbed by the system is (Prosperetti, 1977)

$$P = \frac{-\omega}{2\pi}\int_0^{2\pi/\omega} p_0\left(1+\varepsilon e^{i\omega t}\right)4\pi R^2\dot{R}dt = \frac{4\pi R_0\left(p_0\varepsilon\right)^2\omega^2\beta}{\rho\left[\left(\omega_0^2-\omega^2\right)^2+4\beta^2\omega^2\right]}. \tag{2}$$

and the intensity of the wave is (Landau and Lifchitz, 1971, Ch. 64)

$$I = \frac{\left(p_0\varepsilon\right)^2}{\rho u}. \tag{3}$$

Substituting Eq. (2) and Eq. (3) into Eq. (1) result

$$\sigma = \frac{4\pi R_0\omega^2\beta u}{\left[\left(\omega_0^2-\omega^2\right)^2+4\beta^2\omega^2\right]}. \tag{4}$$

In the above relationships, we have the following notations:

$$R(t) = R_0\left[1+x\left(t\right)\right], \tag{5}$$

with R_0 the equilibrium bubble radius; the amplitude of the incident pressure wave, the forcing wave,

$$p\left(t\right) = \left(p_0\varepsilon\right)e^{i\omega t}, \varepsilon << 1, \tag{6}$$

with p_0 the unperturbed fluid pressure; radial damping constant β which is comprised of the viscous component β_μ, the thermal component β_{th} and the acoustic re-radiated (scattered) component β_{ac}

$$\beta = \beta_\mu + \beta_{th} + \beta_{ac}\,. \tag{7}$$

with

$$\beta_\mu = 2\frac{\mu}{\rho R_0^2},\ \beta_{th} = 2\frac{\mu_{th}}{\rho R_0^2},\ \beta_{ac} = \frac{\omega^2 R_0}{2u} \tag{8}$$

and the natural angular frequency of a radial oscillator

$$\omega_0 = \left[3\gamma\left(\frac{p_0}{\rho R_0^2} + 2\frac{\sigma}{\rho R_0^3}\right) - 2\frac{\sigma}{\rho R_0^3}\right]^{1/2} = \left(\frac{p_{eff}}{\rho R_0^2}\right)^{1/2}. \tag{9}$$

Replacing Eq. (7) in Eq. (4), it result in the shape of the scattering cross section and the absorption cross section

$$\sigma_e = \frac{4\pi R_0\omega^2 u\left(\beta_\mu + \beta_{th} + \beta_{ac}\right)}{\left[\left(\omega_0^2 - \omega^2\right)^2 + 4\beta^2\omega^2\right]} = \frac{4\pi R_0 \beta_{ac}\omega^2 u}{\left[\left(\omega_0^2 - \omega^2\right)^2 + 4\beta^2\omega^2\right]} + \frac{2\pi R_0^2\left(\beta_\mu + \beta_{th}\right)\omega^2 u}{\left[\left(\omega_0^2 - \omega^2\right)^2 + 4\beta^2\omega^2\right]} = \sigma_{ac} + \sigma_a\,. \tag{10}$$

Michael A. Ainslie and Timothy G. Leighton (Ainslie and Leighton, 2011) distinguished three types of the cross sections for the interaction similar to the interaction of electromagnetic waves (light) with a physical system, that maybe a free electron or bound electron: the scattering cross section (re-emission, $\sigma_s = \sigma_{ac}$), the absorption cross section (σ_a) and total cross section (extinction, σ_e). Between all these sections there is relationship $\sigma_e = \sigma_s + \sigma_a$.

Comparing the expressions of the cross sections displayed in this two papers (Ainslie and Leighton, 2011; Prosperetti, 1977) result a difference, observed by Ainslie and Leighton. That is, the correct relation for (4) is

$$\sigma = \frac{8\pi R_0\omega^2\beta u}{\left[\left(\omega_0^2 - \omega^2\right)^2 + 4\beta^2\omega^2\right]} \tag{11}$$

and

$$\sigma_{ac} = \sigma_s = \frac{8\pi R_0\omega^2\beta_{ac} u}{\left[\left(\omega_0^2 - \omega^2\right)^2 + 4\beta^2\omega^2\right]} = \frac{4\pi R_0^2\omega^4}{\left[\left(\omega_0^2 - \omega^2\right)^2 + 4\beta^2\omega^2\right]}. \tag{12}$$

is the scattering cross section.

2.2. The Electrostatic Cross Section

The electrostatic cross section, for a particle with electric charge Q and mass m, oscillating in electromagnetic waves is of the form (Jackson, 1975, Subch. 17.8)

$$\sigma_e = \frac{4\pi R_e \omega^2 \Gamma_t c}{\left[\left(\omega_{0e}^2 - \omega^2\right)^2 + \Gamma_t^{\,2}\omega^2\right]}. \tag{13}$$

with $R_e = Q^2 / \left(4\pi\varepsilon_0 mc^2\right) = e^2 / \left(mc^2\right)$ the electrostatic radius of the particle, $\Gamma_t = \Gamma' + \omega^2 \Gamma / \omega_{0e}^2$ the total decay constant. In previous relationships, Γ' is the absorptive width, $\Gamma / \omega_{0e}^2 = 2e^2 / \left(3mc^3\right) = 2R_e / \left(3c\right)$ is the radiative decay constant, ω_{0e} is the natural angular frequency and ω the angular frequency of the electromagnetic waves.

The electromagnetic scattering cross section is

$$\sigma_{se} = \frac{2}{3} \frac{4\pi R_e^2 \omega^4}{\left[\left(\omega_{0e}^2 - \omega^2\right)^2 + \Gamma_t^{\,2}\omega^2\right]}. \tag{14}$$

By comparing the two cross sections on can establish the following correspondences:

$$R_e \leftrightarrow R_0, \Gamma_t \leftrightarrow 2\beta_r, c \leftrightarrow u, \omega_{0e} \leftrightarrow \omega_0. \tag{15}$$

The difference between (12) and (14) comes from the fact that the two oscillations are different, that of the bubble is radial (the centre of mass remains fixed) and that of the particle one corresponds to a translational motion.

If the particle is free, *i.e.* $\omega_{0e} \to 0$ or $\omega \to \infty$ then the cross section (14) becomes the Thomson cross section (Jackson, 1975, Subch. 14.7), which is independent of the angular frequency of the incident radiation

$$\sigma_T = \frac{2}{3}\left(4\pi R_e^2\right) = \frac{8\pi}{3}\left(\frac{e^2}{m_e c^2}\right)^2. \tag{16}$$

The cross section for the acoustic scattering resembles the Thomson cross section in the limit cases, $\omega_0 \to 0$ or $\omega \to \infty$ and at resonance.

In the limit cases, the acoustic scattering cross section is

$$\sigma_{Bac} = \sigma_{Bs} = \lim_{\omega_0 \to 0} \sigma_s = \lim_{\omega \to \infty} \sigma_s = 4\pi R_0^2. \tag{17}$$

At speed resonance, $\omega_{rez} = \omega_0$ (Feynman *et al*., 1964, Ch. 23), the acoustic scattering cross section (12) becomes

$$\sigma_{0ac} = \sigma_{0s} = 4\pi R_0^2 \left(\frac{\rho u^2}{p_{eff}} \right). \tag{18}$$

Analogues to the Thomson section, they do not depend on the angular frequency.

2.3. The Electrostatic Force as Function on the Thomson Cross Section

The electrostatic force between two electrons can be expressed according to the Thomson cross section (Eq. (16) (Jackson, 1975, Subch. 14.7)). To obtain this expression, we calculate the average energy density of the classical electron treated as a sphere with radius

$$R_e = \frac{e^2}{m_e c^2}, \tag{19}$$

according to the relationship

$$w_e = \frac{m_e}{V_e} = \frac{3m_e c^2}{4\pi R_e^3} \tag{20a}$$

or

$$m_e = \frac{4\pi R_e^3}{3c^2} w_e . \tag{20b}$$

Replacing Eq. (20b) into Eq. (19) and taking into account Eq. (16), we obtain

$$e^2 = \frac{4\pi}{3} R_e^4 w_e = \frac{3}{16\pi} \sigma_T^2 w_e . \tag{21}$$

Therefore, the expression of the Coulomb force for the electron in vacuum (Jackson, 1975, Ch.1), $F_{Ce}(r) = q_e^2 / \left(4\pi\varepsilon_0 r^2\right) = e^2/r^2$, becomes

$$F_{Ce}(r) = \frac{\sigma_T^2}{8\pi r^2}\left(\frac{3w_e}{2}\right) = \frac{\sigma_T^2}{8\pi r^2}\left(\frac{9p_e}{2}\right),\ p_e = \frac{w_e}{3} . \tag{22}$$

The expression of the Coulomb force (22) and the expression of the resonant acoustic force $F_B(\omega_0, r) \cong \left[\sigma_{0s}^2 / \left(8\pi r^2\right)\right]\left[\left(p_0\varepsilon\right)^2 / \left(\rho u^2\right)\right]$ (Eq. (6) from paper (Simaciu *et al*., 2019) or Eq. (24) from (Simaciu *et al*., arXiv: 1711.03567)), become similar only in the case when the pressure under which bubbles interact is a constant of the system. We emphasize the system consisting of a fluid in a container and bubbles in fluid.

2.4. The Acoustic Cross Section in a Thermal Acoustic Background

We are entitled to make the assumption that the acoustic cross section for the radial movement becomes a cross section of the Thomson type. This can be shown if one calculates the average cross section of all frequencies of the thermal background corresponding to the acoustic radiation of the container in which the bubble is:

$$\sigma_{Ta} = \sigma_B = \frac{\int_{\omega_m}^{\omega_M} \sigma(\omega)\rho(\omega)d\omega}{\int_{\omega_m}^{\omega_M} \rho(\omega)d\omega} = \frac{4\pi R_0^2 \int_{\omega_m}^{\omega_M} \frac{\omega^7 d\omega}{\left[\exp\left(\hbar\omega/(kT)\right)-1\right]\left[\left(\omega_0^2-\omega^2\right)^2+4\beta^2\omega^2\right]}}{\int_{\omega_m}^{\omega_M} \frac{\omega^3 d\omega}{\exp\left[\hbar\omega/(kT)\right]-1}}. \quad (23)$$

We remind that the thermal background is the acoustic radiation of the container which contains the bubbles.

The same result is reached when in the definition (1) of the cross section it is inserted the power and the intensity calculated by the averaged random-phase according to the section 3.1. from the paper (Simaciu *et al.*, 2019).

2.5. The Acoustic Cross Section in the Background

Now we will estimate analytically the expression (19) of the acoustic cross section in the background. To do this we calculate the integral using the saddle-point method (Simaciu *et al.*, 2019; Puthoff, 1987; Feynman *et al.*, 1964, Ch. 23).

$$I_{\sigma 1} = \int_{\omega_m}^{\omega_M} \frac{\omega^7 d\omega}{\left[\exp\left(\hbar\omega/(kT)\right)-1\right]\left[\left(\omega_0^2-\omega^2\right)^2+4\beta^2\omega^2\right]} = \frac{\omega_0^5}{4\left[\exp\left(\hbar\omega_0/(kT)\right)-1\right]}\int_{y_m}^{y_M}\frac{-dy}{y^2+\beta_0^2} \cong \frac{\pi\omega_0^5}{8\beta_0\left[\exp\left(\hbar\omega_0/(kT)\right)-1\right]} \cong \frac{\pi\omega_0^3}{4R_0\left[\exp\left(\hbar\omega_0/(kT)\right)-1\right]}. \quad (24)$$

The second integral is

$$I_{\sigma 2}=\int_{\omega_m}^{\omega_M}\rho(\omega)d\omega=\int_{\omega_m}^{\omega_M}\left[\frac{\hbar\omega^3}{\pi^2u^3}\frac{1}{\exp\left[\hbar\omega/(kT)\right]-1}\right]=$$

$$\left(\frac{kT}{\hbar}\right)^4\int_{x_m}^{x_M}\frac{x^3dx}{\exp x-1}\cong\left(\frac{kT}{\hbar}\right)^4\int_0^{\hbar\omega_M/(kT)}\frac{x^3dx}{\exp x-1}\cong \tag{25}$$

$$\left(\frac{kT}{\hbar}\right)^4\int_0^{\infty}\frac{x^3dx}{\exp x-1}\cong\frac{\pi^4}{15}\left(\frac{kT}{\hbar}\right)^4.$$

Replacing the two integrals (24) and (25) into Eq. (23), it results

$$\sigma_B=\frac{15R_0\omega_0^3u}{\pi^2\left(\frac{kT}{\hbar}\right)^4\left[\exp\left(\hbar\omega_0/(kT)\right)-1\right]}. \tag{26}$$

Depending on the ratio between $\hbar\omega_0$ and kT, we get the following expressions of the scattering cross section:

$$\sigma_{B1}=\frac{15\hbar^3R_0\omega_0^2u}{\pi^2(kT)^3}=\frac{15}{\pi^2R_0}\left(\frac{\hbar u}{kT}\right)^3\left(\frac{p_{eff}}{\rho u^2}\right),\hbar\omega_0<<kT; \tag{27}$$

$$\sigma_{B2}=\frac{15\hbar^4R_0\omega_0^3u}{\pi^2(kT)^4(e-1)}=\frac{15}{\pi^2(e-1)}\left(\frac{\hbar u}{kT}\right)^2\left(\frac{p_{eff}}{\rho u^2}\right)^{\frac{1}{2}},\hbar\omega_0\cong kT; \tag{28}$$

$$\sigma_{B3}=\frac{15\hbar^4R_0\omega_0^3u}{\pi^2(kT)^4\exp\left(\hbar\omega_0/(kT)\right)}=\frac{15u^2}{\pi^2\omega_0^2\exp\left(\hbar\omega_0/(kT)\right)}\left(\frac{p_{eff}}{\rho u^2}\right)^{1/2}=$$

$$\frac{15R_0^2}{\pi^2\exp\left(\hbar u\sqrt{p_{eff}}/\left(R_0kT\sqrt{\rho u^2}\right)\right)}\left(\frac{\rho u^2}{p_{eff}}\right)^{1/2},\hbar\omega_0>>kT. \tag{29}$$

Analyzing the expressions of Eqs. (26-29), we note that the relations (28) are weak dependent of the bubble parameters through the effective pressure, Eq. (9). It results that the expressions of the scattering -absorption cross section also emphasize the existence of a maximum acoustic charge, $e_{am}^2=\hbar u$, (according to Eq. 54 of the paper (Simaciu *et al*., 2019)).

3. Acoustic Field

By analogy with the electrostatic field (Jackson, 1975, Subch. 1.2), the acoustic charge, at rest, produces a static acoustic field of intensity $\vec{E}_a$:

$$\vec{E}_a = \frac{\vec{F}_a}{e_a} = \frac{e_a}{r^3}\vec{r}\ , \tag{30}$$

The energy density of this acoustic field is

$$w_a\left(r\right) = \frac{1}{8\pi}\vec{E}_a^2 = \frac{e_a^2}{8\pi r^4}\ . \tag{31}$$

If we consider the bubble with radius R_a as a system that scatters acoustic waves, the acoustic energy density on the scattering surface is

$$w_a\left(R_a\right) = \frac{e_a^2}{8\pi R_a^4}\ . \tag{32}$$

By analogy to electrostatics, the equivalent mass of this system is

$$m_a = V_a\frac{w_a\left(R_a\right)}{u^2} = \frac{4\pi R_a^3 w_a\left(R_a\right)}{3u^2} = \frac{e_a^2}{6u^2R_a}\ . \tag{33}$$

This mass is related to the mass corresponding to the acoustic field up to a constant. This mass is different from the virtual mass of the bubble $m_{vb} = \left(2/3\right)\pi R_0^3\rho$ (Landau *et al*., 1971, Ch. 1, 3). The mass of the acoustic energy of the field generated by the bubbles is

$$m_{fa} = \frac{1}{u^2}\int_{R_a}^{L} w_a 4\pi r^2 dr = \frac{e_a^2}{2u^2R_a} - \frac{e_a^2}{2u^2L} \cong \frac{e_a^2}{2u^2R_a}, L >> R_a\ . \tag{34}$$

where L is the radius of the container.

If in Eq. (33) we consider the last equality (34), then it follows the relationship

$$e_a^2 = 8\pi R_a^4 w_a\left(R_a\right) = \frac{\sigma_a^2}{2\pi}w_a\left(R_a\right), \tag{35}$$

with $\sigma_a = \sigma_B = 4\pi R_a^2$. The relationship (35) is analogous to the electrostatic relationship given by Eq. (21).

From the equality $\sigma_a = \sigma_B$ and Eq. (26), one can express the radius of the system which scatter the acoustic waves, through the oscillating bubble parameters:

$$R_a = \left(\frac{\hbar u}{kT}\right)^2\left[\frac{15R_0\omega_0^3}{4\pi^3u^3\left[\exp\left(\hbar\omega_0/\left(kT\right)\right)-1\right]}\right]^{1/2}\ . \tag{36}$$

In the particular case given by the condition of Eq. (28), a simple relationship follows

$$R_a = \frac{R_0}{2\pi}\left[\frac{15}{\pi(e-1)}\left(\frac{p_{eff}}{\rho u^2}\right)^{1/2}\right]^{1/2}, \tag{37}$$

that is, the acoustic radius is proportional to the bubble radius and the liquid parameter.

4. Conclusions

In this paper we have shown that there is also an analogy between the electromagnetic interaction and the acoustic interaction between two bubbles in the liquid from the point of view of the scattering-absorption cross section.

For identical bubbles, we identified the existence of an acoustic cross section. The acoustic extinction cross section, which has the definition in Eq. (1) and having the expression (4), does not depend on the amplitude of the forcing wave but depends on the properties of the bubble. The bubble is assumed to be a forced radial oscillator with damping process and with the centre of the bubble at rest. The scattering cross section of the free electron, according to Eq. (16) is independent of the angular frequency. The cross section and the acoustic charge, are not related of the angular frequency for angular frequency close to the natural angular frequency that is at resonance. In this case, the two parameters remain dependent on the magnitude of the forcing wave. In order to eliminate the two dependencies we had two options to consider: the two bubbles interact with the background of the thermal radiation or the two bubbles interact with the background created by other identical oscillating bubbles in the container. Adopting the first option, we have obtained an acoustic charge and a scattering cross section analogous to electrostatic ones.

REFERENCES

Ainslie M.A., Leighton T.G., *Review of Scattering and Extinction Cross-Sections, Damping Factors, and Resonance Frequencies of a Spherical Gas Bubble*, J. Acoust. Soc. Am., **130**, *5*, Pub. 12667 (2011).

Feynman R.P., Leighton R.B., Sands M., *The Feynman Lectures on Physics*, Vol. **1**, Addison-Wesley, Reading, 1964.

Jackson J.D., *Classical Electrodynamics*, Second Edition, Wiley, New York, 1975.

Landau L., Lifchitz E., *Mecanique des fluids*, Edition Mir, Moscou, 1971.

Prosperetti A., *Thermale Effects and Damping Mechanisms in the Forced Radial Oscillations of Gas Bubbles in Liquids*, J. Acoust. Soc. Am., **61**, *1*, Pt. 2, Ianuary, 17-27 (1977).

Puthoff H.E., *Ground State of Hydrogen as a Zero-Point-Fluctuation-Determined State*, Phys. Rev. D, **35**, 3266-3269 (1987).

Simaciu I., Borsos Z., Dumitrescu Gh., Silva G.T., Bărbat T., *Acoustic Scattering-Extinction Cross Section and the Acoustic Force of Electrostatic Type*, arXiv: 1711.03567.

Simaciu I., Borsos Z., Dumitrescu Gh., Silva G.T., Bărbat T., *The Acoustic Force of Electrostatic Type*, Bul. Inst. Polit. Iaşi, s. Matematică. Mecanică Teoretică. Fizică, **65 (69)**, *2*, 15-28, (2019).

SECȚIUNEA ACUSTICĂ DE ÎMPRĂȘTIERE ȘI ABSORBȚIE DE TIP ELECTROSTATIC

(Rezumat)

Analiza forței secundare Bjerknes între două bule sugerează că această forță este analogă forțelor electrostatice. Aceeași analogie este sugerată și de existența unei secțiuni acustice de împrăștiere-absorpție (de extincție) a unei unde acustice pe o bulă. În această lucrare analizăm analogia dintre secțiunea de împrăștiere-absorbție a unei unde acustice pe o bulă din lichid și împrăștierea-absorbția undelor electromagnetice de către o sarcină electrică.